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# 1 Equipment Acquired

The following table lists items requested and estimated costs together with items acquired and actual costs. The rationale is given for all variations. All changes were made in order to enhance performance at the same cost.

<u>Items Requested</u>	<u>Estimated Cost</u>	<u>Items Acquired</u> (Rationale for variation) [Manufacturer's name]	<u>Actual Cost</u>
1. Supercomputer	\$ 71,997	Challenge L [Silicon Graphics] (Better compute server, the upgrade to R8000 processors increased cpu performance 3-5 times over Crimson RE. Graphics and video editing capabilities provided by Indigo <sup>2</sup> Extreme)	87,421
2. Laser Printer	\$ 8,496	two (2) HP 1200 CPS (color postscript) one (1) HP 550 C (Lower cost per page than Phaser IISD \$0.25 vs. \$4.00 per page allowing greater access to printers.) [Hewlett Packard]	6,851
3. Disk Drive	\$ 5,741	Barracuda disk drive [Barracuda] same drive - 1/2 the cost	2,130
4. Tape Drive (2)	\$ 3,838	Included with Challenge L, see item 1	
5. Software	\$ 6,783	Licensed from TSL (The Software Library) at Washington University. Item 26 is included here.	7,708
6. Drive (2)	\$ 2,355	Included in Indigo <sup>2</sup> Extremes (see item 14)	
7. Network Interface (8)	\$ 10,166	10 - Based - T Hub [Volksnet] (Installation of 10-based-T network simplified the setup and management of the computers, thereby reducing the need for network interfaces)	740
8. Software (4)	\$ 5,995	Software for SGI	6,994
9. Software	\$ 496	Phaser IID items replaced by those listed in item 2.	
10. Memory Upgrade	\$ 421		
11. Phaser print (4)	\$ 1,020		
12. Transceiver	\$ 145	Ethernet cable	90
13. Transceiver cable	\$ 150		
14. Workstations (3)	\$ 51,932	Two (2) Indigo <sup>2</sup> Extremes [Silicon Graphics] (better graphics performance than the XZ which was requested. Also, has video editing option) Two (2) NCD X-Terminals [Network Computing Devices] (provided more seats for user base.)	75,625

15. Video	\$ 13,000	(Not purchased;	
16. VideoEditor/ Animator Controller	\$ 2,796	video editing capabilities provided by Indigo <sup>2</sup> Extreme, see item 14)	
17. Video-in-Window Card	\$ 938	(Not purchased;	
18. Accelerator	\$ 247	video editing capabilities provided by	
19. Generator	\$ 379	Indigo <sup>2</sup> Extreme, see item 14)	
20. Color Video Monitor	\$ 966	Two (2) PVM-1344Q Monitors [Sony]	1,960
21. Editing VCR	\$ 14,263	Two (2) S-VHS VCR's [Sony]	4,810
22. Source VCR	\$ 9,493	(S-VHS was preferred over Beta	
23. Cable (2)	\$ 186	due to limited availability of Beta recorders)	
24. Quadra 800 (3)	\$ 14,466	Two (2) Macintosh 840 AV [Macintosh] Macintosh 660AV [Macintosh] HP 9000 model 715 [Hewlett Packard] (The higher performance Macintoshes were obtained at the same total cost. The programs used by Dr. Szabo could not run on the Macintosh because of insufficient computational and OS resources . The funds allocated for 1 Quadra were used to purchase an HP workstation. The Macintosh 660 AV was funded from money saved in other areas.)	23,742
25. LaserWriter	\$ 1,839	HP 4M PostScript Laser Printer [Hewlett Packard]	2,220
26. Software (4)	\$ 1,600	Maple (see item 5)	
27. Matlab Software (4)	\$ 2,780	Matlab for SGI	2,650
28. Matlab Software for Mac	\$ 990	Not needed, using SGI version	
29. Compilers (4)	\$2,800	Included inTSL software (see item 5)	
31. Setup Cost	\$5,000	FrameMaker Upgrades (software) 2,594 HP LaserJet 4M[Hewlett Packard] 2,220 Bernoulli System for Macintosh 948 Color Risc Xterminal [Network Computing Devices] 2,823 Macintosh memory upgrades 1,811 Textures (software) 517 Modem [Macintosh] 261 Security Systems 102 Computer Cables 133 (2) Seagate Disk Drives 5,976	17,385
Total Funds Awarded	<u>\$241,278</u>	Total Funds Used	<u>\$240,326</u>

## 2 Research Projects Using the Equipment

The equipment purchased under this grant has been (is being) used on the following research projects:

- 2.1 "Nonlinear Control Systems," Principal Investigator: Professor C.I. Byrnes, AFOSR Grant No. 91-0266, Grant Monitor, Dr. Marc Q. Jacobs (202) 767-5025.**

(\$287,508 8/15/91-10/14/94)

**"Doctoral Research in Systems Science and Mathematics," (EPSCoR Grant), Principal Investigator: Professor C.I. Byrnes, AFOSR Grant No. 91-0266, Grant Monitor, Dr. Marc Q. Jacobs (202) 767-5025.**

(\$119,446 8/15/92-8/14/95)

### 2.1.1 Control of Finite Dimensional Nonlinear Systems

#### 2.1.1.1 Numerical Solution of Nonlinear PDE's in Nonlinear Control

In recent years, it has been discovered that an impressive array of feedback design problems for nonlinear control systems can be solved in terms of the solution of certain systems of nonlinear PDE's. This is very close in spirit to the situation for linear control system design, where it is now well known that a variety of basic design problems can be solved in terms of a hierarchy of matrix equations: the quadratic (nonlinear) Riccati equation, the Sylvester equation and its more simple form, the Lyapunov equation. Basic and important problems of nonlinear control theory, such as feedback linearization, output regulation, optimal control and  $H_\infty$  control, are all now known to be solvable if, and only if, certain PDE's are solvable. The most well-known example is the HJB PDE for optimal control, which is one nonlinear form of the matrix Riccati equation.

Another example is what is now known as the FBI (Francis-Byrnes-Isidori) partial differential equation for the problem of output regulation, which in the linear case reduces to the Sylvester equation discovered by Francis. In the nonlinear setting, this has been applied to some research problems of direct interest to the DoD.

The nonlinear control research group in the Department of Systems Science and Mathematics at Washington University has applied nonlinear  $H_\infty$  theory to the problem of aircraft control under windshear during taking-off. Considerable numerical testing has been carried out and it has been shown that the performance of this nonlinear  $H_\infty$  control system compares favorably with the many existing results found in the literature. In particular, several advantages of incorporating a quadratic term in the feedback law are noted, including an enlarged domain of attraction and better steady state response.

The numerical solution of the HJI PDE in the problem of aircraft control through windshear via  $H_\infty$  methods is only a part of a large framework concerning PDE's arising in nonlinear control theory. Nearly all basic design problems in nonlinear control rely on the development of efficient methods for the numerical solutions of the related hierarchy of partial differential equations. There are two approaches to the numerical solution of the PDE's arising in nonlinear control that we have studied. One is based on the homological equations approach to higher order approximations, pioneered by Krener in his development of the POINCARÉ method, and one based on finite difference and finite element approximations.

In either of the two research directions, a huge amount of computational work on the computer is unavoidable for simulations and off line computation. The higher degree approximation methods involve the solution of large algebraic systems with many parameters. Software developed before acquiring the equipment on this grant, could only compute the quadratic part of the solutions to HJI PDE. The computation for cubic terms has now been made possible with the aid of the powerful computer system, appropriate computational software and graphics subsystems obtained on this grant. The supercomputing equipment, capable of high quality graphics and video animation capabilities, has been used extensively in this research effort.

#### 2.1.1.2 Semiglobal Robust Output Regulation

We are currently investigating the semiglobal version of structurally stable synthesis. More specifically, we are separately addressing two issues. The first one is to establish conditions under which the problem of *local* output regulation has a solution for every value of a possibly unknown parameter in a *a priori fixed* compact set in the parameter space. The second issue is to be able to assure that the problem of *semiglobal* output regulation has a solution for every value of this parameter in a *a priori fixed* compact set.

The results of our preliminary analysis show that the study of the first problem may yield quite elegant, frequency-domain based, conditions for the existence of a controller securing output regulation. We have carried out several numerical experiments in this area with suprising success. In particular, we have considered output regulation for a controlled van der Pol oscillator containing a parameter in a fixed compact set. This system is driven by a harmonic oscillator. Using an error feedback we were able to choose design parameters in such a way as to achieve robust stability of the closed loop system.

#### 2.1.2 Boundary Control of Nonlinear Distributed Parameter Systems

One of our longer term goals as part of this research project, is the development of a systematic feedback design methodology for nonlinear distributed parameter systems which would retain some of the intuitive appeal of classical automatic control in a spirit similar to the program pursued for the control of lumped nonlinear systems. In this area we have made extensive use of the scientific computing equipment obtained under this grant. In particular, the acquisition of this equipment has now made it possible for us to carry out simulations far beyond our earlier expectations. In what follows we describe briefly several areas in which this equipment has been used.



### 2.1.2.1 Convergence of trajectories for nonlinear DPS

Our interests in the control of nonlinear distributed parameter systems include the development of systematic strategies for designing feedback laws which can shape or at least influence the response of nonlinear distributed parameter systems.

For example, under appropriate hypotheses on a controlled Burgers' equation, we have proven that as we tune a gain parameter  $k$ , the closed-loop trajectories approach the trajectories of the zero dynamics. In particular, in this problem the zero dynamics has an attractor consisting of a single global asymptotically stable equilibrium. Using our analytic results on convergence of trajectories we have shown that the local attractor of the closed loop system, which varies with the gains, converge to the attractor of the zero dynamics. We have carried out many numerical simulations depicting this phenomenon and have used the graphical visualization equipment to document this convergence in the form of pictures that depict this motion.

Both analytically and numerically, we have been able to show that the dynamics of the closed loop system subject to unknown disturbance can be rather complicated. Along these line we have been able to numerically demonstrate that depending on the values of the gain parameters in our closed loop Burgers' system, there may be multiple stationary points which, of course, suggests that the local attractor is not trivial. Since it is not possible to construct these stationary solutions explicitly, the control systems computer equipment has been an indispensable tool in our analysis.

### 2.1.2.2 Regulation and Tracking

Another of our long term goals is the development of a design methodology capable of shaping the response of distributed parameter systems. Among the most important problems in control theory is that of controlling a fixed plant in order to have its output track (or reject) reference (or disturbance) signals produced by some external generator (the exosystem). The local solution of this problem is characterized in terms of the solvability of a system of partial differential equations, the "regulator equations." Moreover, there is a geometric criterion for the solvability of the regulator equations in terms of the system zero dynamics and the zero dynamics of the exogenous system. This geometric existence criterion often is known to be automatically satisfied by appealing to invariant manifold results from nonlinear dynamics and its expression in terms of zero dynamics allows for a nonlinear enhancement of classical tracking criteria involving linear transmission zeros. Finally, the structure of the regulator equations permit a refinement of the equations which yield a rather familiar form of a feedback law achieving output regulation; viz., the superposition of a stabilizing law and a feedthrough term, which both depend upon a nonlinear gain function which itself can be determined off-line by solving a nonlinear PDE.

We are interested in the problem of output regulation for distributed parameter systems in the sense described above for lumped nonlinear systems. As in this case, we expect the main ingredients in the solution of this problem to be the design of stabilizing feedback laws and the development of a theory of steady state response for stable distributed parameter systems.

In this direction, we have used the stabilizing boundary feedback law discussed above to stabilize a non-asymptotically stable open loop Burgers' system. This exponentially stable closed loop system is then driven through the boundary by the output of a neutrally stable finite dimensional exogenous system. In an extensive numerical investigation of this model system we have discovered that such periodic boundary forcing, indeed, produces asymptotically periodic output. This numerical

evidence strongly supports our expectations that it will be possible to solve the regulatory problem for a large class of nonlinear distributed parameter systems.

**2.2 "Research and Development of the p-version of the Finite Element Method," Principal Investigator: Professor I. Norman Katz, Co-Principal Investigator: Professor Barna A. Szabo, AFOSR Grant No. F49620-92-J-0043, Grant Monitor, Dr. Marc Q. Jacobs (202) 767-5025.**

(\$512,458 11/15/91-11/14/94)

**"Doctoral Research in Systems Science and Mathematics and Mechanical Engineering (AASERT Grant)," Principal Investigator: Professor I. Norman Katz, Co-Principal Investigator: Professor Barna A. Szabo, AFOSR Grant No. 88-0017, Grant Monitor, Dr. Marc Q. Jacobs (202) 767-5025.**

(\$180,212 6/1/92-5/31/95)

### **2.2.1 Parallel Implementations of the p-version of the Finite Element Method**

An iterative method based on the textured decomposition has been developed in order to solve the systems of linear equations arising in the  $p$ -version of the finite element method. The iteration is used to implement the  $p$ -version in parallel. The objectives are two-fold: to achieve high computational efficiency (that is computational load should be balanced among the processors) and simultaneously to achieve rapid convergence.

A superelement, consisting of four adjacent rectangular finite elements, has been constructed for two dimensional problems. Based on the structural property of the shape functions, each superelement is partitioned into three blocks in two different ways, and a two-leaf textured decomposition (TD) is used. Computations for a superelement associated with each leaf are assigned to two processors and are performed in parallel. A new preconditioner is introduced to accelerate convergence in a preconditioned textured decomposition (PTD). A special local communication strategy is used to avoid global assembly and global communication.

Three model problems: a Poisson equation on a rectangular domain with a smooth true solution, a Laplace equation on a rectangular domain with a near singular solution, and a Poisson equation on L-shaped domain, are solved. The conjugate gradient method, the textured decomposition method, the recursive textured decomposition method, (both with and without preconditioning); and the classical iterative methods (Jacobi, Gauss-Seidel, SOR), are used to solve the three model problems. Load balance, speedup ratio, and spectral radii of the various iterations are studied. The test results indicate the recursive PTD with a local communication strategy gives at least a 30% improvement in computational time over the other methods.



### 2.2.2 Multi- $p$ methods: Iterative algorithms for the $p$ -version of the finite element analysis

Motivated by classical multigrid methods, which are based on either finite difference method or the  $h$ -version of the finite element method, we are studying multi- $p$  methods, which are based on the  $p$ -version of the finite element method and hierarchical shape functions. We have developed a class of multi- $p$  V-cycle algorithms: standard V-cycle, modified V-cycle and varying V-cycle. We have also studied nested multi- $p$  methods. By combining the nested multi- $p$  methods with the V-cycle methods, we obtain the so-called "accelerated multi- $p$  V-cycle methods". The convergence of multi- $p$  V-cycle methods have been investigated and an error estimate is given for the nested multi- $p$  methods.

Numerical experiments on representative sample problems have been conducted which indicate that our multi- $p$  methods are very efficient.

### 2.2.3 Algebraic Multi- $p$ methods and Multi- $p$ preconditioners

An algebraic theory for multi- $p$  methods is formulated and analyzed. Convergence and symmetric properties are proved under suitable conditions. It is then shown how these multi- $p$  methods can be used as preconditioners for the conjugate gradient method (CG). In particular, it is shown that given any preconditioner  $M_p$  to CG, a multi- $p$  preconditioner  $B_p$  based on  $M_p$  can be constructed, which leads to a small condition number (and hence faster convergence). When  $B_p$  is applied as a preconditioner to condensed finite elements, the condition number is shown to grow slower than  $C(1 + \log^2 p)$ , the best currently known result for 2-D problems in the  $p$ -version of the finite element analysis.

Numerical experiments on representative problems indicate that the condition numbers after multi- $p$  preconditionings are, in fact, independent of  $p$ . The numerical results also show greater efficiency for PCG with the multi- $p$  preconditioners in terms of number of iterations and CPU time when compared with two sophisticated linear equation solvers: (1) a direct frontal solver specially designed for the  $p$ -version of the finite element analysis; (2) a highly tuned preconditioned CG code in ITPACK. Preliminary comparisons of the number of iterations are also made with ROCKITS, a new commercial code used for the  $p$ -version.

The methods presented are intended for use in the  $p$ -version of the finite element analysis, but are general in nature and can be applied to a wide variety of problems.

## 2.3 "Advanced Mathematical Models for Structural Systems," Principal Investigator Professor Barna A. Szabo, DARPA grant number F49620-93-1-0173, Grant Monitor, Dr. Marc Q. Jacobs (202) 767-5025.

(\$734,953 2/15/93-2/14/96)

### 2.3.1 Convergence of Stress Maxima in Finite Element Computations

The convergence of stress maxima, computed directly from finite element solutions, has been investigated with respect to a family of exact solutions characterized by varying degrees of smoothness. The performances of  $h$ - and  $p$ -extensions and the product and trunk spaces are evaluated and documented with respect to a family of benchmark problems. In uniform  $p$ -extensions a characteristic pattern in the convergence of stress maxima was observed. There does not appear to be a clear-cut advantage of the product space over the trunk space in this respect. The much faster convergence of stress maxima in the case of  $p$ -extensions, as compared with  $h$ -extensions, is evident from the results.

### 2.3.2 PEGASYS: A Software Infrastructure for R & D

PEGASYS is an advanced software infrastructure, designed with two objectives in mind: To support research and to provide a framework for technology deployment in numerical simulation.

Essentially, PEGASYS is an open system of software modules with well-documented interface specifications. PEGASYS currently consists of approximately 250,000 lines of code written in the FORTRAN and C languages. PEGASYS is maintained in the popular workstations, such as Silicon Graphics. Using the software modules of PEGASYS, it is possible for researchers and developers to assemble simulation software products designed to meet specific engineering and scientific objectives.

A number of finite element procedures concerned with element formulation, mapping, assembly, extraction procedures, error estimation and adaptivity have been completed. The currently active R&D projects utilizing PEGASYS include the following:

- Development of hierarchic models for structural plates and shells;
- Development of hierarchic models for laminated composites;
- Superconvergent methods for the computation of stress intensity factors, including generalized stress intensity factors for multi-material interface problems;
- Investigation of crack propagation phenomena;
- Mechanics of filament-wound composites;
- Advanced solution methods for elastic-plastic problems;
- Development of models for structural connections;
- Error estimation and quality control procedures;
- Implementation of advanced finite element software on parallel computers;
- Design sensitivity analysis.

### 2.3.3 Investigation of mechanical problems characterized by large displacements and small strains

This investigation is concerned with modelling large displacement responses of structural beams, plates and shells. The large displacement response is important in flutter problems and structural stability problems. A new methodology, based on the assumption of a linear relationship between the Cauchy stresses and the Almansi strains, and utilizing the  $p$ -version of the finite element method, promises a rapid and robust convergence for a wide range of practical problems.

## 2.4 Summary

All of the research described here requires extensive computer calculations. The equipment which we have acquired under this grant has impacted all three projects by providing additional computing power, high-level graphics capabilities, and the opportunity to produce professional-level videos.